Nervous Presidents – Revision

Note Title 11/12/2012

States

- the position of the boat (2 possibilities)
- lB the number of bodyguards on the left bank
- lP the number of presidents on the left bank.

$$State = BoatPosition \times NoBGsOnLeft \times NoPsOnLeft$$

$$BoatPosition = \{Left, Right\}$$

$$NoBGsOnLeft = \{0..N\}$$

$$NoPsOnLeft = \{0..N\}$$

$$|State| = 2 \times (N+1) \times (N+1)$$

Valid States

A president cannot be with a bodyguard unless his/her bodyguard is also present.

ℓB : NoBGsOnLeft

ℓP : NoPsOnLeft

$$\ell B = 0 \;\vee\; \ell B = N \;\vee\; \ell B = \ell P$$

The number of states satisfying $(\ell B = \ell P)$ is $N+1$.

The number of states satisfying

$$(\ell B = 0 \;\vee\; \ell B = N) \wedge \neg(\ell B = \ell P) \text{ is } 2\times N.$$

Hence, total no. of *valid* states $= 2\times(N+1+2\times N) = 6\times N + 2$.

Effect of (Unnecessary) Naming

Suppose couples are named $1..N$.
presidents are named $P1 \ldots PN$.
bodyguards are named $B1 \ldots BN$.

- the position of the boat (2 possibilities)
- ℓB the set of bodyguards on the left bank
- ℓP the set of presidents on the left bank.

State = BoatPosition × SetOfBGsOnLeft × SetOfPsOnLeft

BoatPosition = {Left, Right}

SetOfBGsOnLeft = $2^{\{1..N\}}$

SetOfPsOnLeft = $2^{\{1..N\}}$

$$|\text{State}| = 2 \times 2^N \times 2^N$$

$$|\ell B| = 0 \quad \vee \quad |\ell B| = N \quad \vee \quad |\ell B| = |\ell P|$$

$$\ell B = \emptyset \quad \vee \quad \ell B = \{1..N\} \quad \vee \quad \ell B = \ell P$$

Rewrite as

$$\ell B = \emptyset \quad \vee \quad \ell B = \{1..N\} \quad \vee \quad \emptyset \subset \ell B = \ell P \subset \{1..N\}$$

Total no. of *valid* states $= 2^N + 2^N + (2^N - 2)$

$= 3 \times 2^N - 2.$

Conclusion

Avoid unnecessary naming.

An inappropriate representation of a problem can lead to an explosion in the size of the state space.